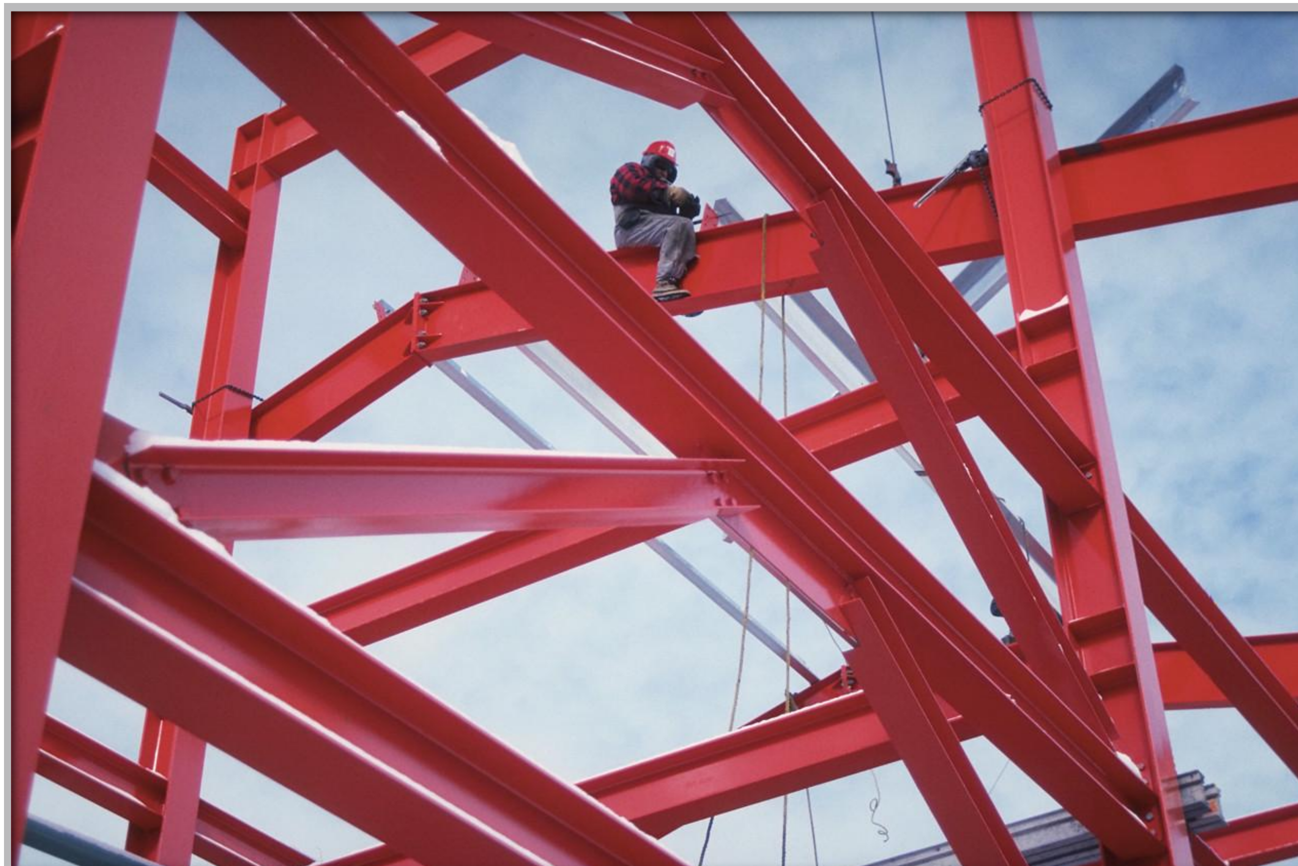
Mechanical Vibrations

VIBRATIONS

Fundamentals concepts



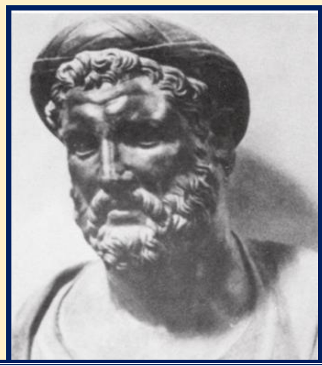
Eng. Laith Batarseh

Fundamentals concepts

VIBRATIONS

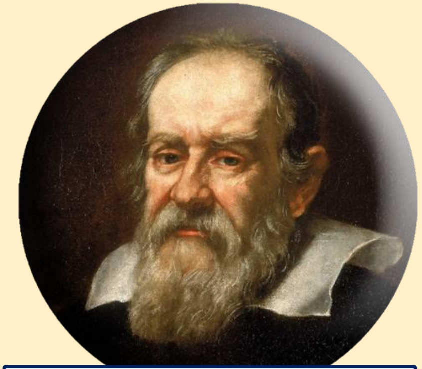
- **Brief History**

- **Vibration study started from about 6000 years ago when the 1st musical instrument was invented.**



Pythagoras (582-507 B.C)

Zhang Heng (132 A.D)

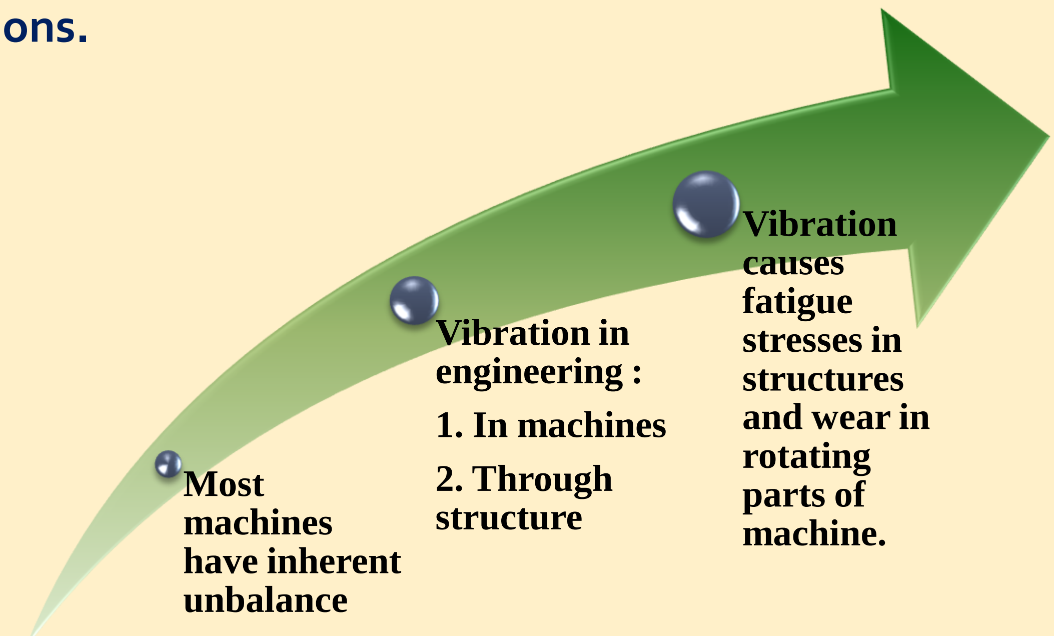


Galileo Galilee (1564-1642)

Fundamentals concepts

Importance of the study of vibration

- Vibration is founded all around us
- In past, vibration was studied to understand the physical phenomena and derive a mathematical model to describe it .
- In recent times, the motivation of studying vibration is the engineering applications.

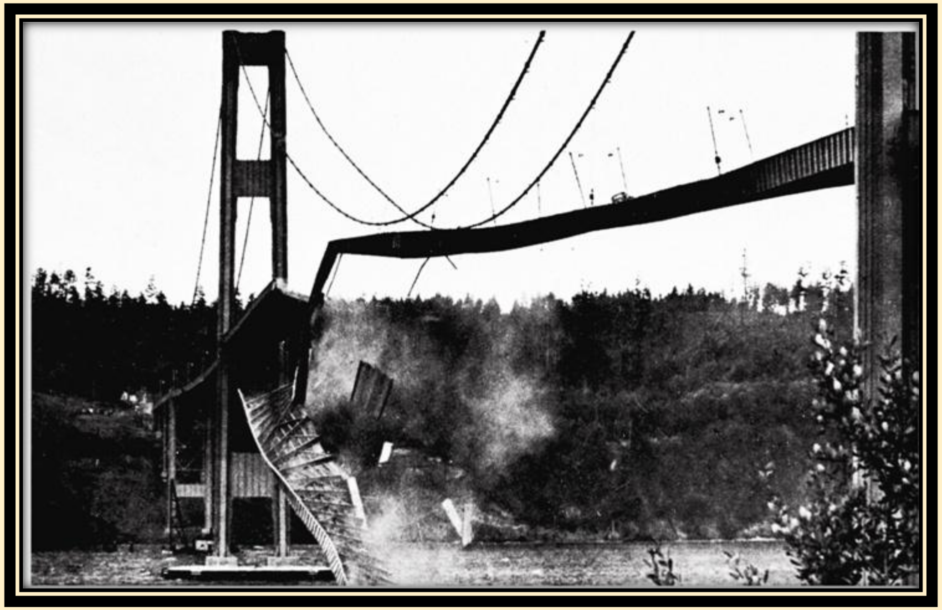


Fundamentals concepts

Importance of the study of vibration

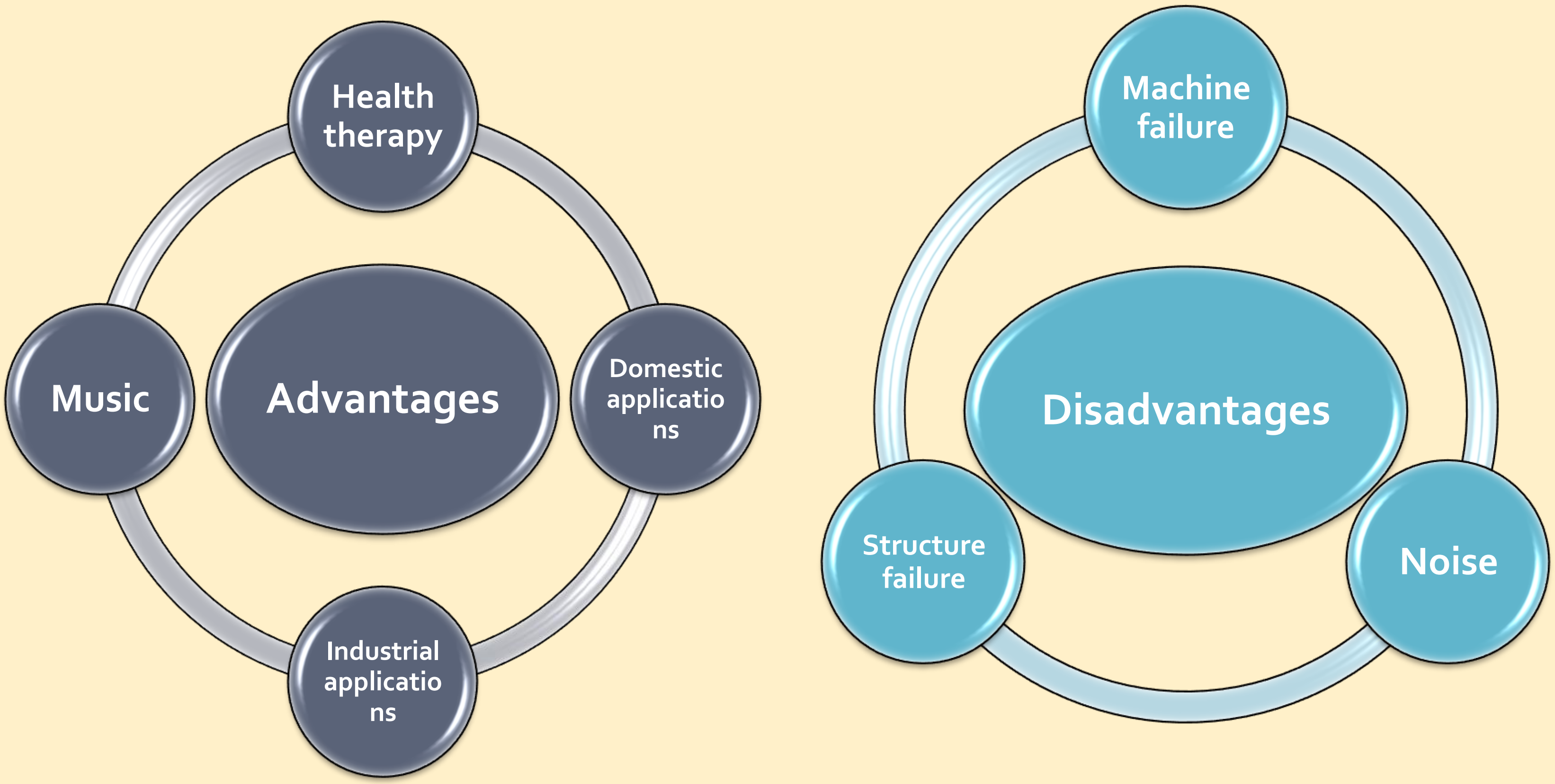
- Resonance is one of the most devastating effects of vibration on machines and structures.
- Resonance happen when the natural frequency of the system equals the excitation frequency of the external excitation.

Tacoma Narrows bridge failure due to wind excitation (July 1,1940 – November 7,1940)



Fundamentals concepts

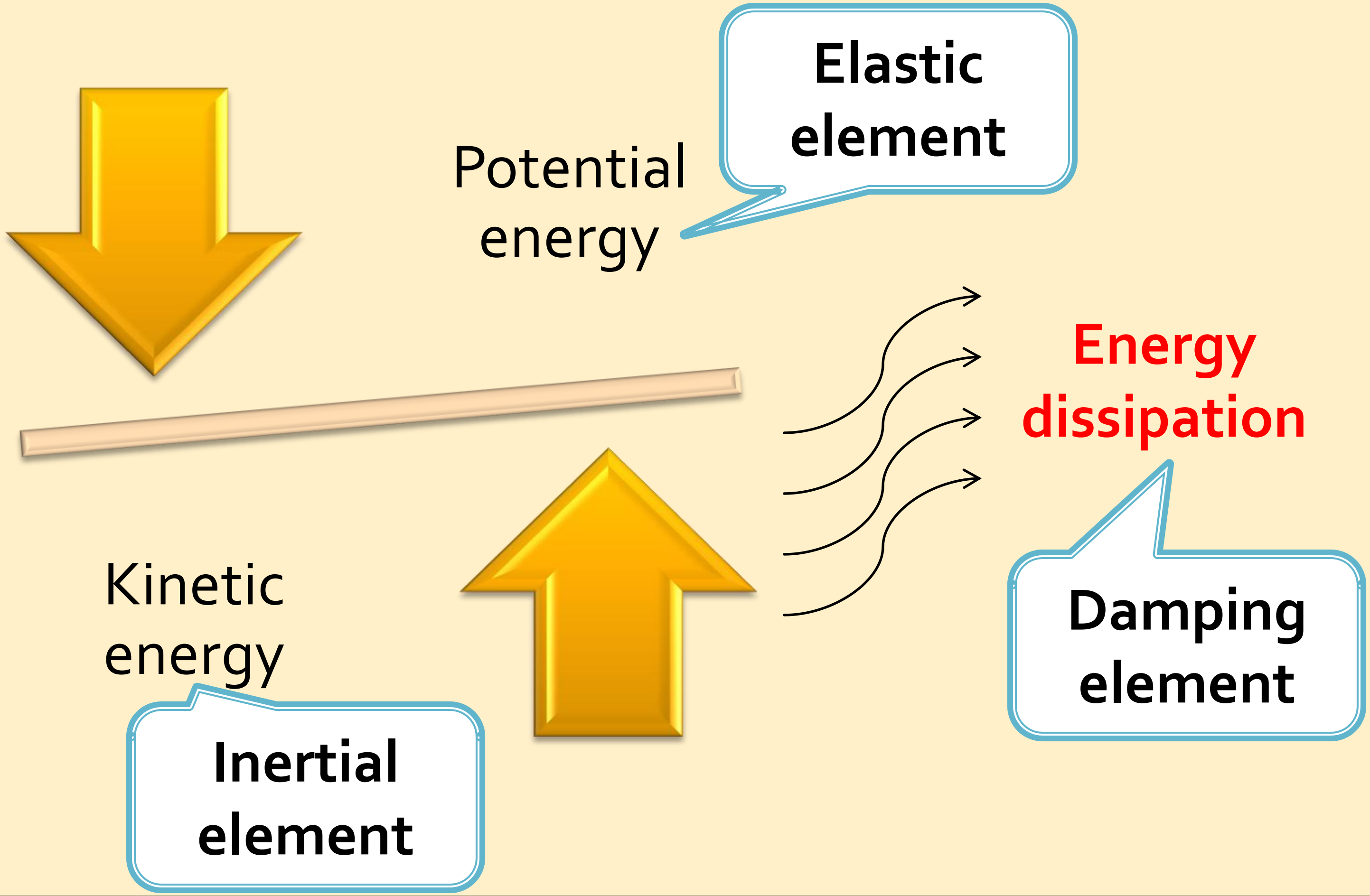
- Importance of the study of vibration



VIBRATIONS

Fundamentals concepts

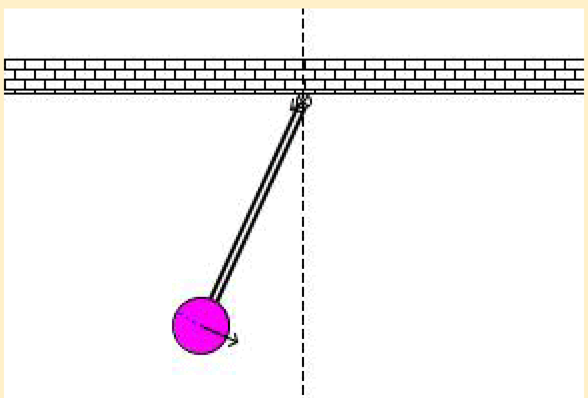
- **Vibration system elementary parts**

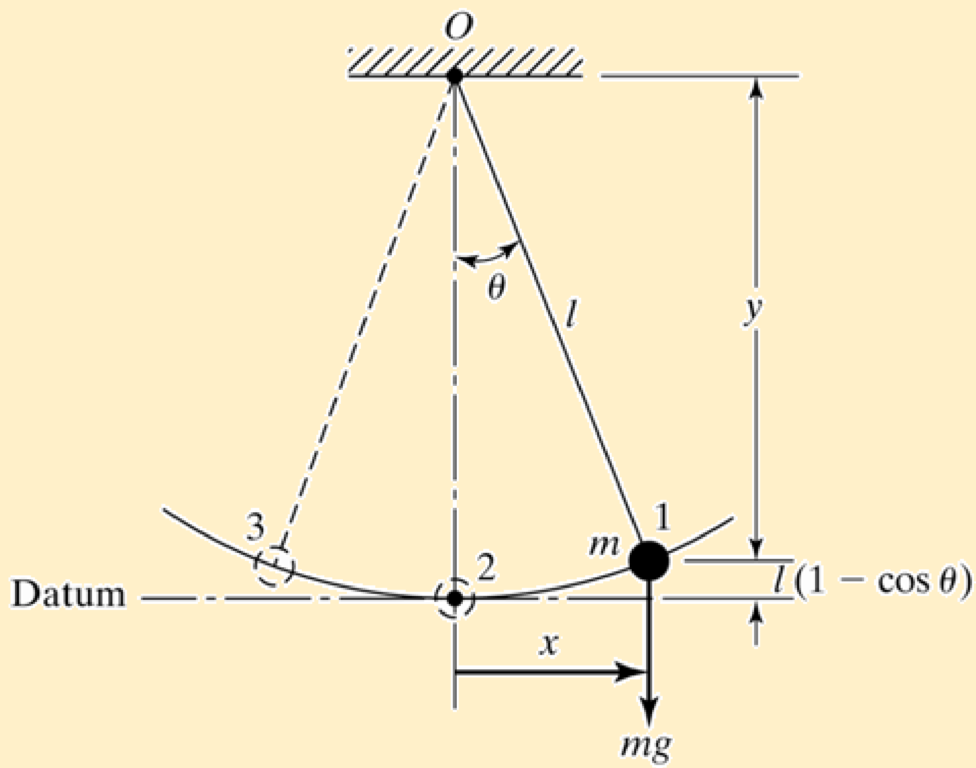


Fundamentals concepts

- **Examples**

Rotational vibration

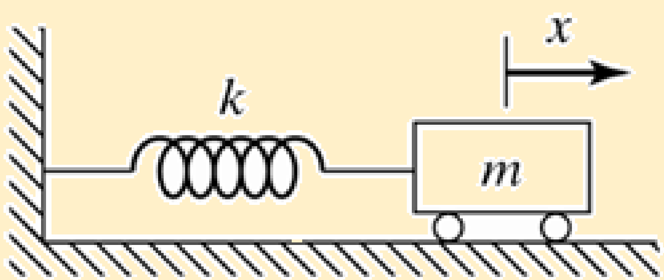


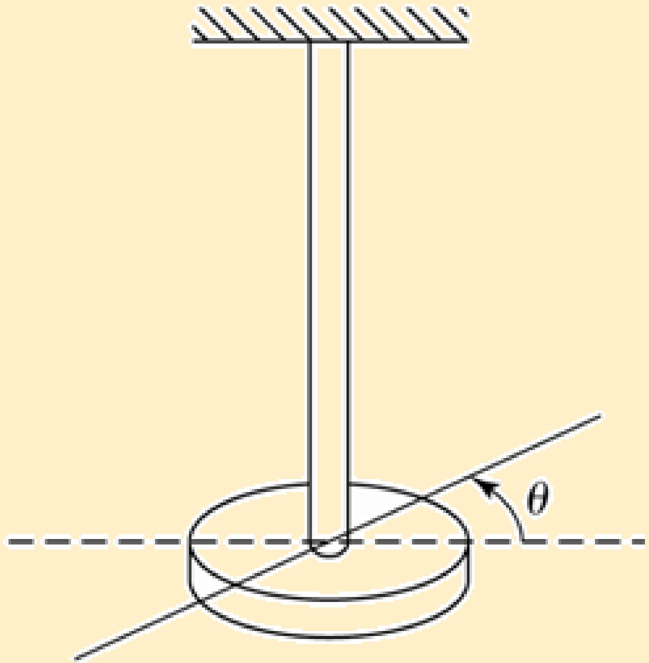


Fundamentals concepts

- Other examples

Translational vibration





Toriosnal vibration

Fundamentals concepts

- **Degree of Freedom (DoF)**

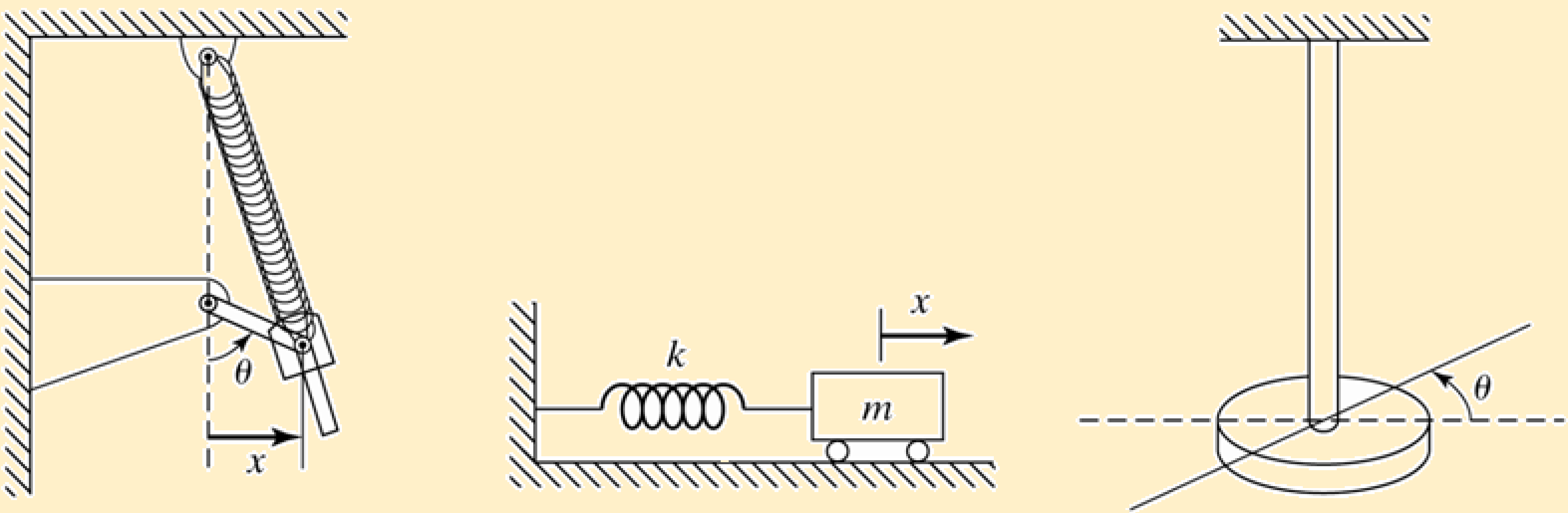
Definition

The minimum number of independent coordinates required to determine completely the positions of all parts of the system at any instant.

Fundamentals concepts

- **Degree of Freedom (DoF)**

Examples (single DoF)



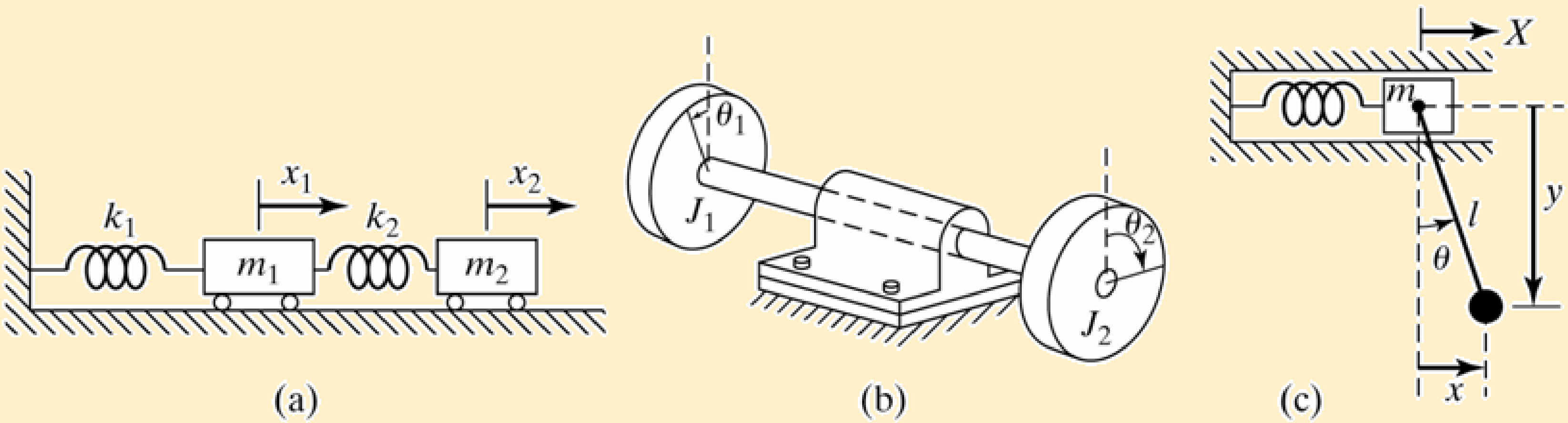
(a) Slider-crank-spring mechanism (b) Spring-mass system (c) Torsional system

Fundamentals concepts

V I B R A T I O N S

- Degree of Freedom (DoF)

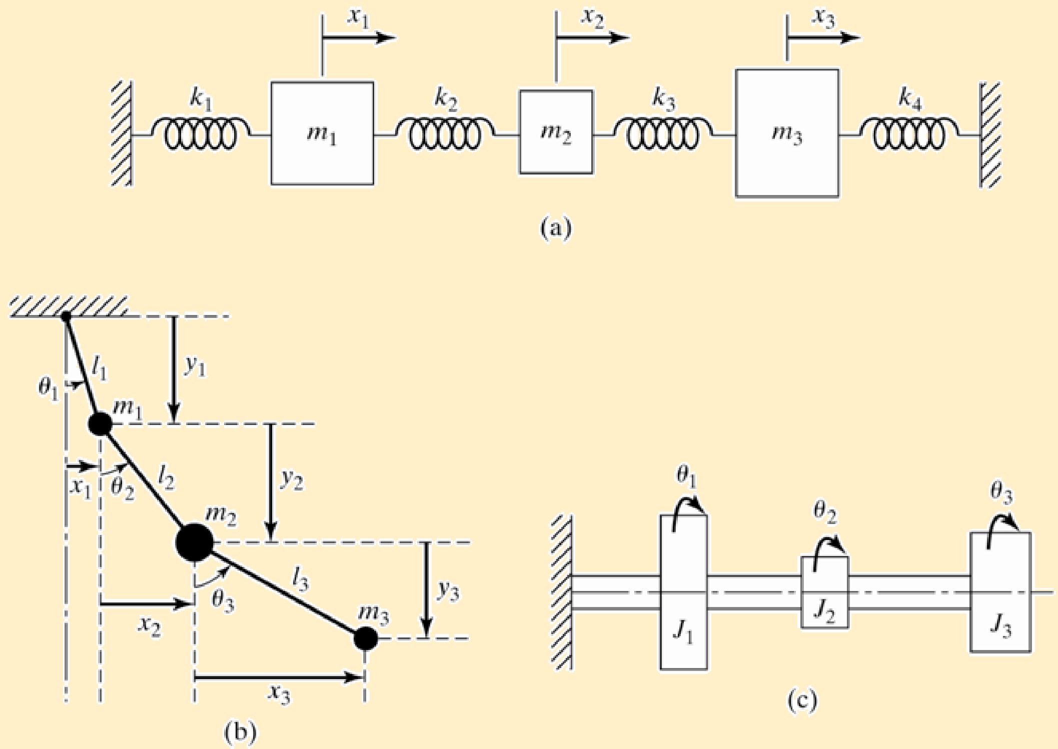
Examples (Two DoF)



Fundamentals concepts

- Degree of Freedom (DoF)

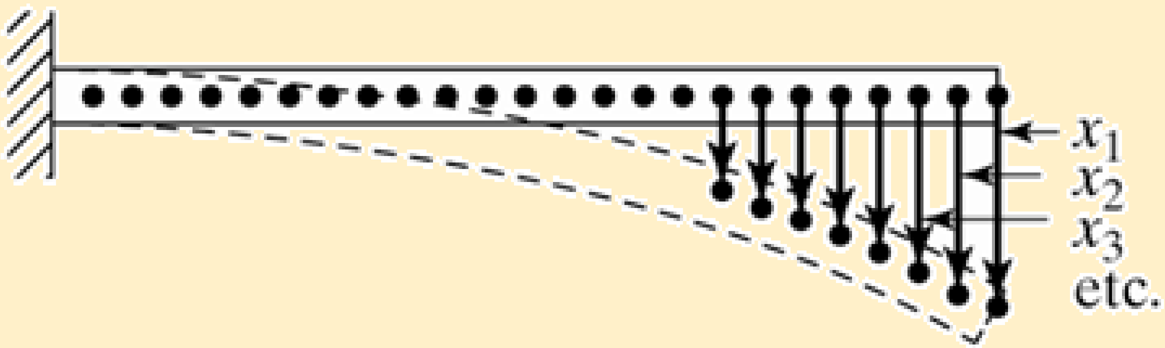
Examples (Three DoF)



Fundamentals concepts

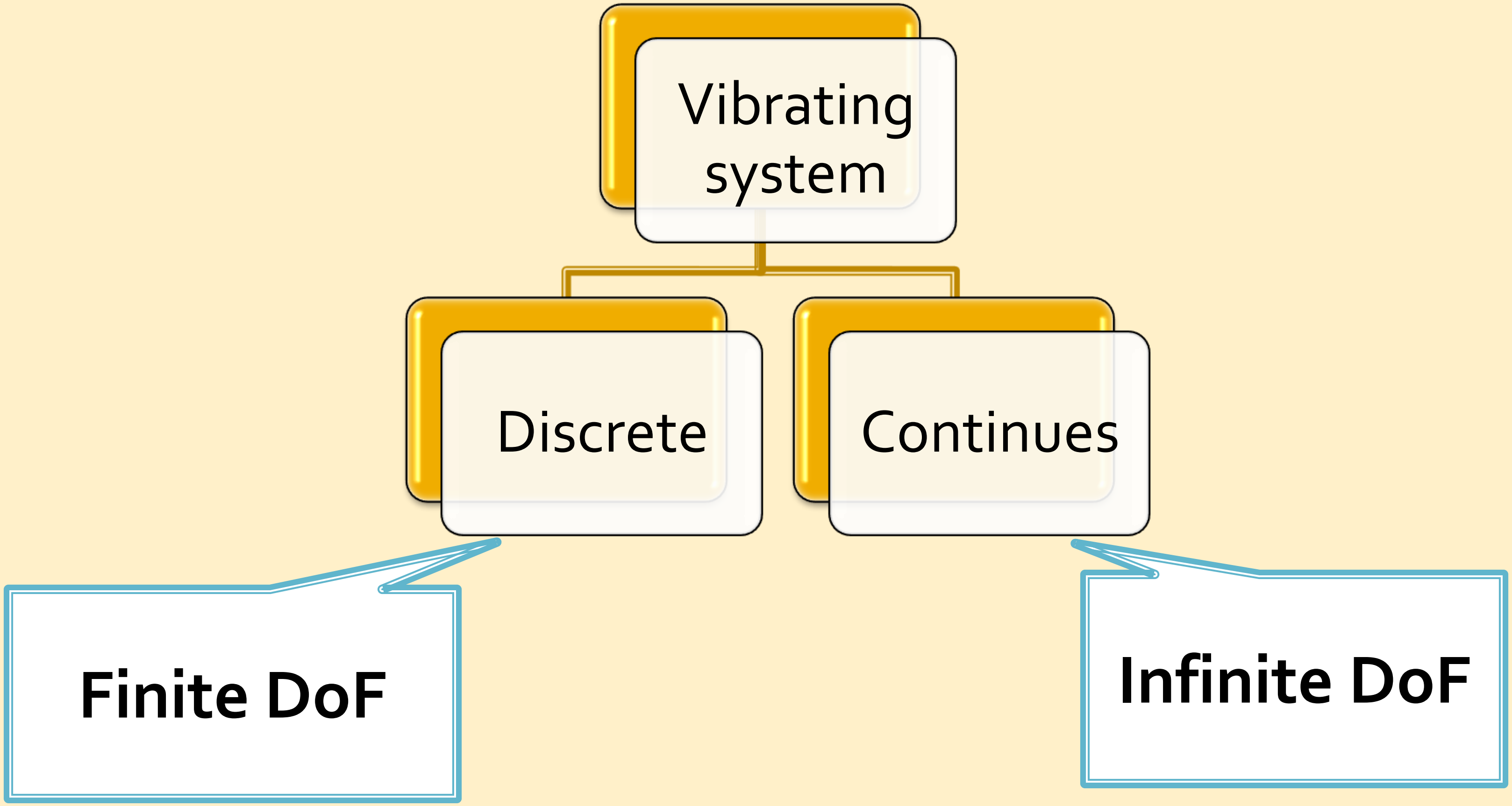
- **Degree of Freedom (DoF)**

Examples (infinite DoF)



Fundamentals concepts

- Discrete and continues systems



Fundamentals concepts

- Vibration classifications

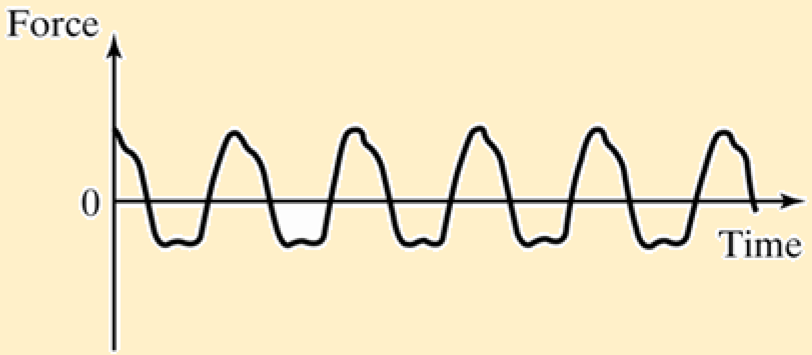
Vibration

- **Excitation**
  - **Free**: Applying initial excitation only and leave the system to vibrate
  - **Forced**: Continues repeating force acting on the system
- **Damping**
  - **Damped**: Energy dissipation after a period of time
  - **Un-damped**: No energy dissipation and system will vibrate for ever
- **Linearity**
  - **Linear**: All the basic components (spring, mass and damper) all have linear behaviors
  - **None-linear**: If any of the vibration system elements act nonlinearly
- **Determent**
  - **Deterministic**: If the magnitude of excitation is known at any time
  - **Random**: If the magnitude of excitation can not be predicted at a given time

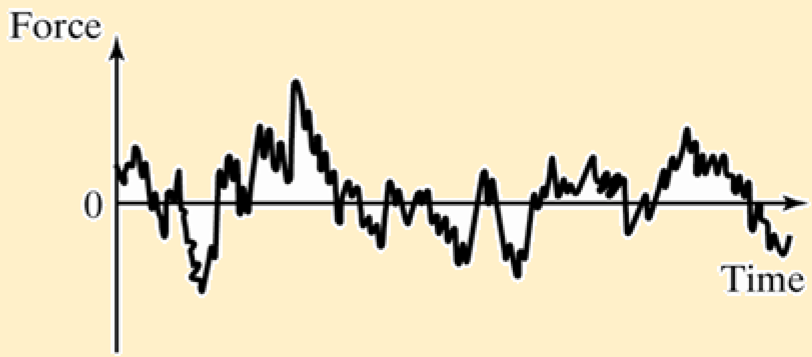
Fundamentals concepts

V I B R A T I O N S

- **Deterministic and random vibrations**



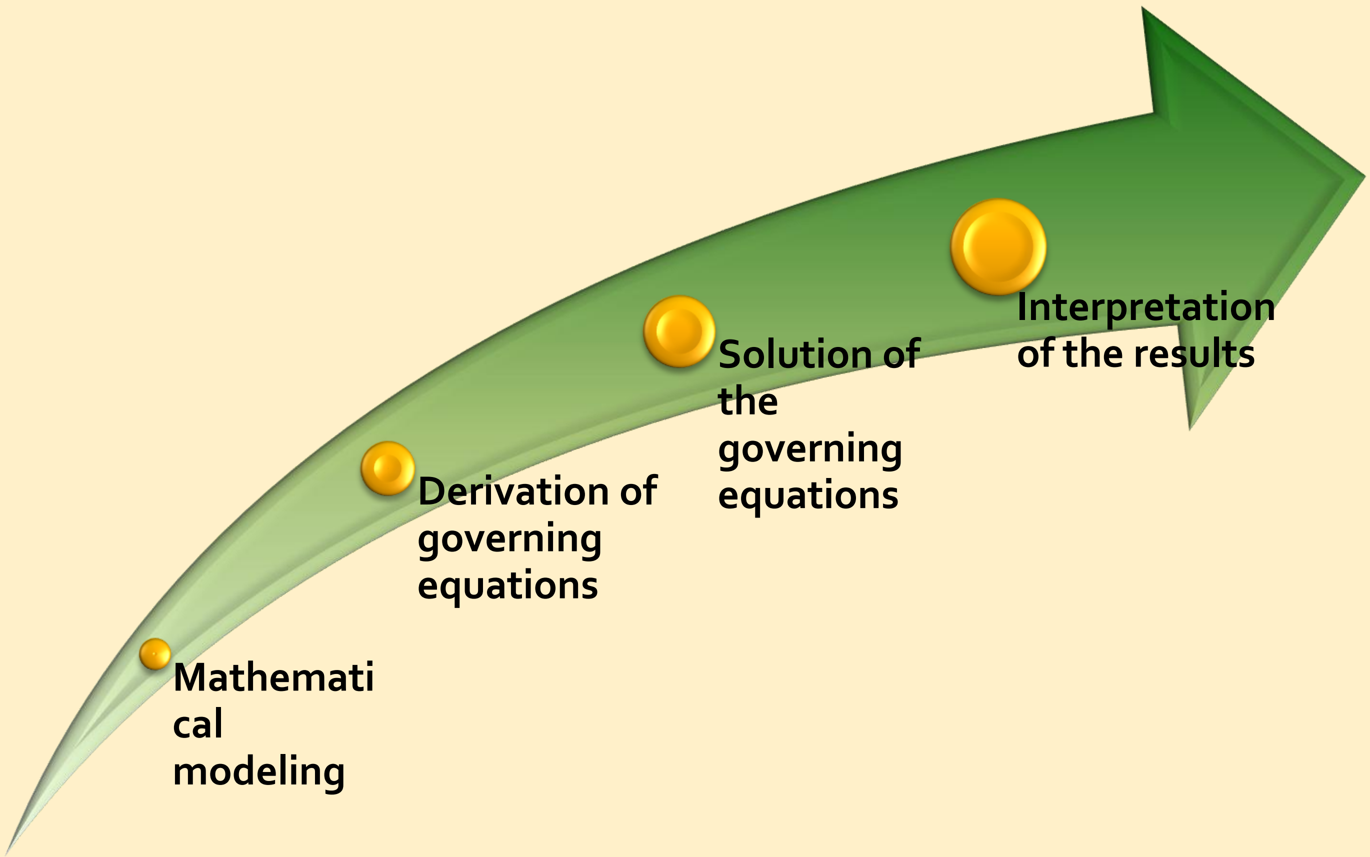
(a) A deterministic (periodic) excitation



(b) A random excitation

Fundamentals concepts

Vibration analysis procedures



Fundamentals concepts

Vibration system elements

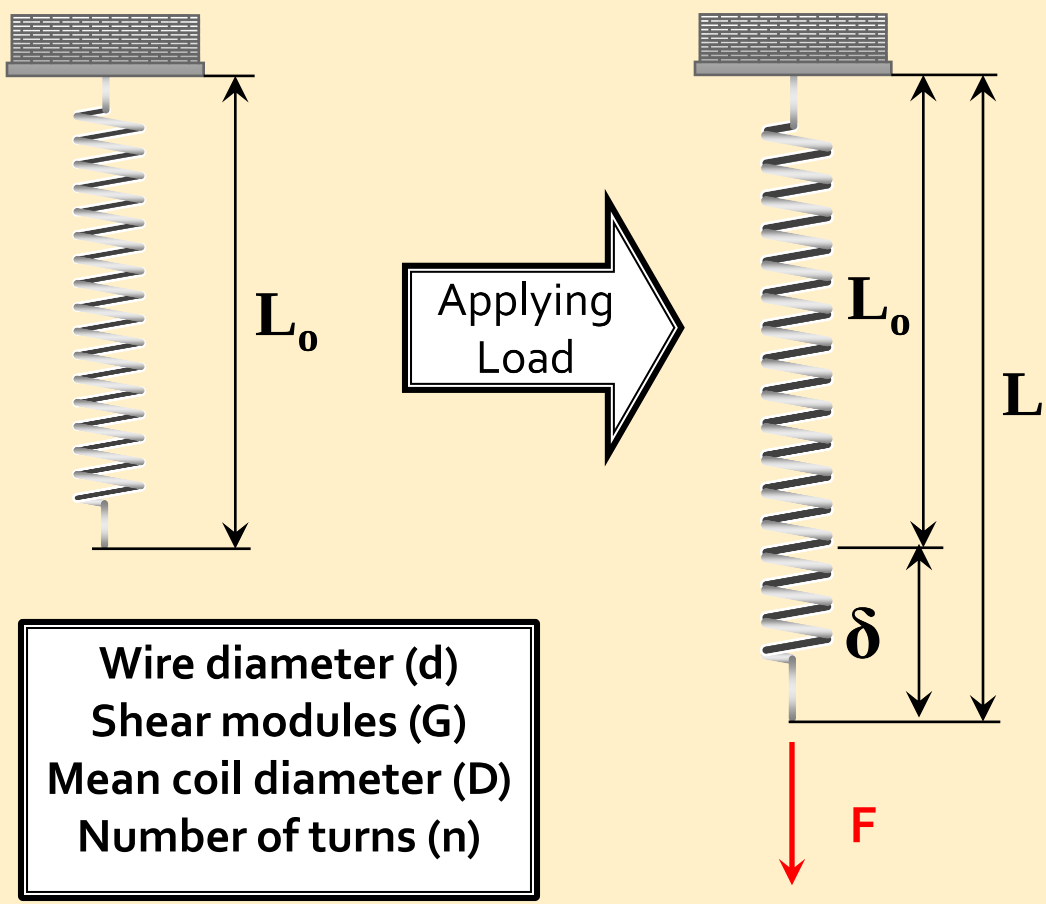
Vibration system elements

Spring	Inertia	Damper
Has stiffness (k)	Has mass (M,I)	Has damping effect (C)
Neglected mass and damping effect.	Neglected stiffness and damping effect.	Neglected mass and stiffness

Fundamentals concepts

STATICS

Vibration system elements : spring



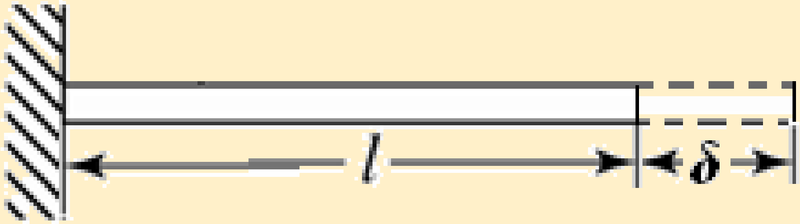
Wire diameter (d)
Shear modules (G)
Mean coil diameter (D)
Number of turns (n)

- deflection $(\delta) = L - L_o$
- $F = K\ \delta$ where K is the spring stiffness
- increasing K makes the spring stiffer.
- Stiffer springs needs more force to deflect it
- Potential energy (U): $U = 0.5\ k.x^2$
- $K = (d^4G/8D^3n)$

Fundamentals concepts

S T A T I C S

Spring special case: rod



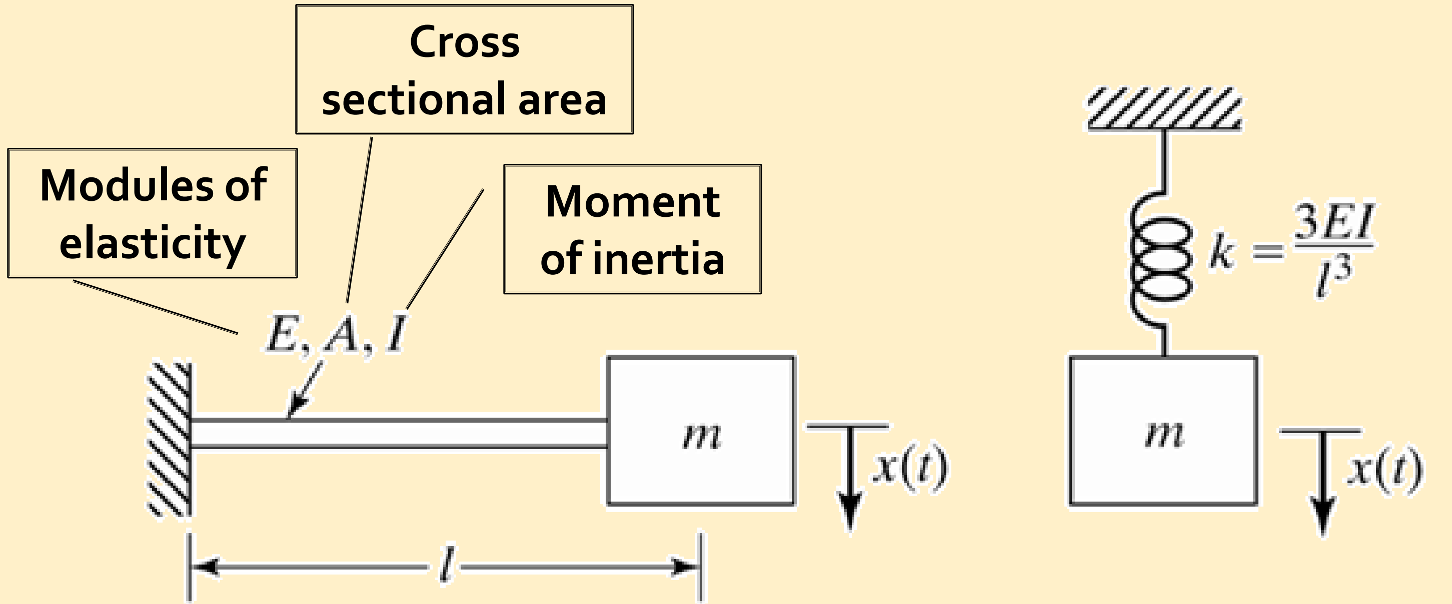
$$k = \frac{AE}{l}$$

Cross sectional area (A)

Modules of elasticity (E)

Fundamentals concepts

Spring special case: cantilever beam



(a) Actual system

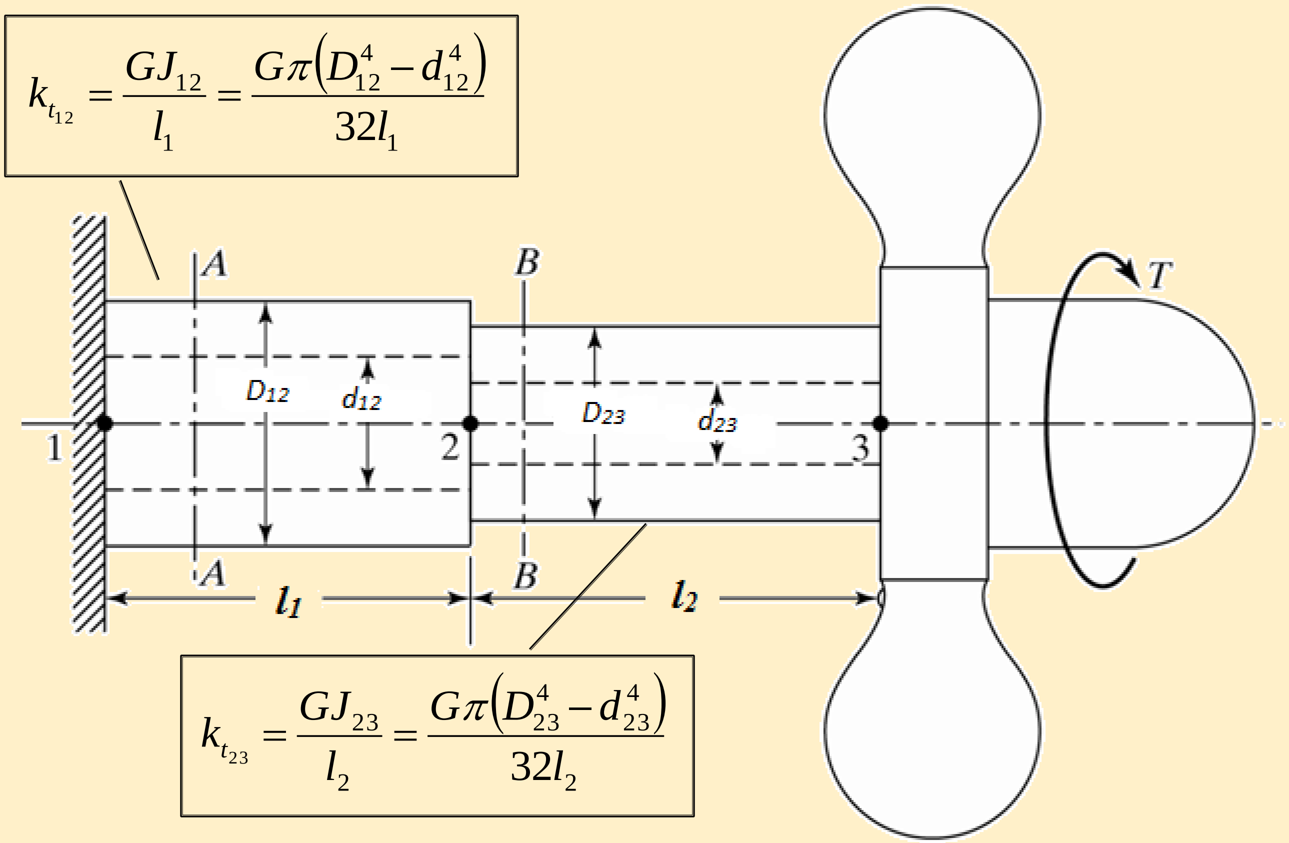
(b) Single degree of freedom model

S T A T I C S

Fundamentals concepts

Spring special case: torsional stiffness of shafts

$$k_{t_{12}} = \frac{GJ_{12}}{l_1} = \frac{G\pi\left(D_{12}^4 - d_{12}^4\right)}{32 l_1}$$

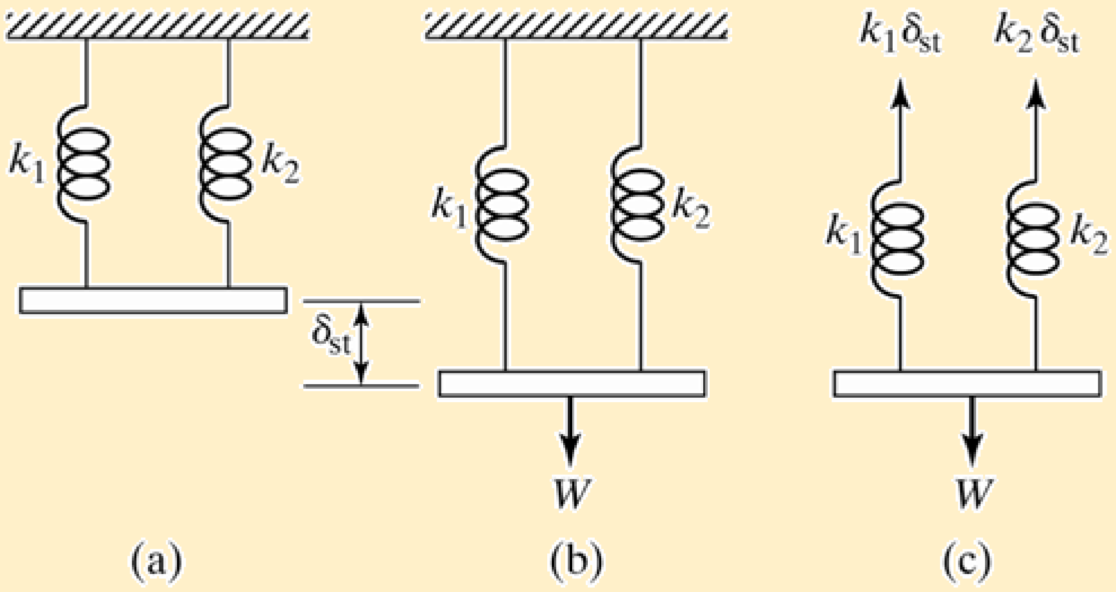


$$k_{t_{23}} = \frac{GJ_{23}}{l_2} = \frac{G\pi\left(D_{23}^4 - d_{23}^4\right)}{32 l_2}$$

Fundamentals concepts

S T A T I C S

Spring special case: parallel connection

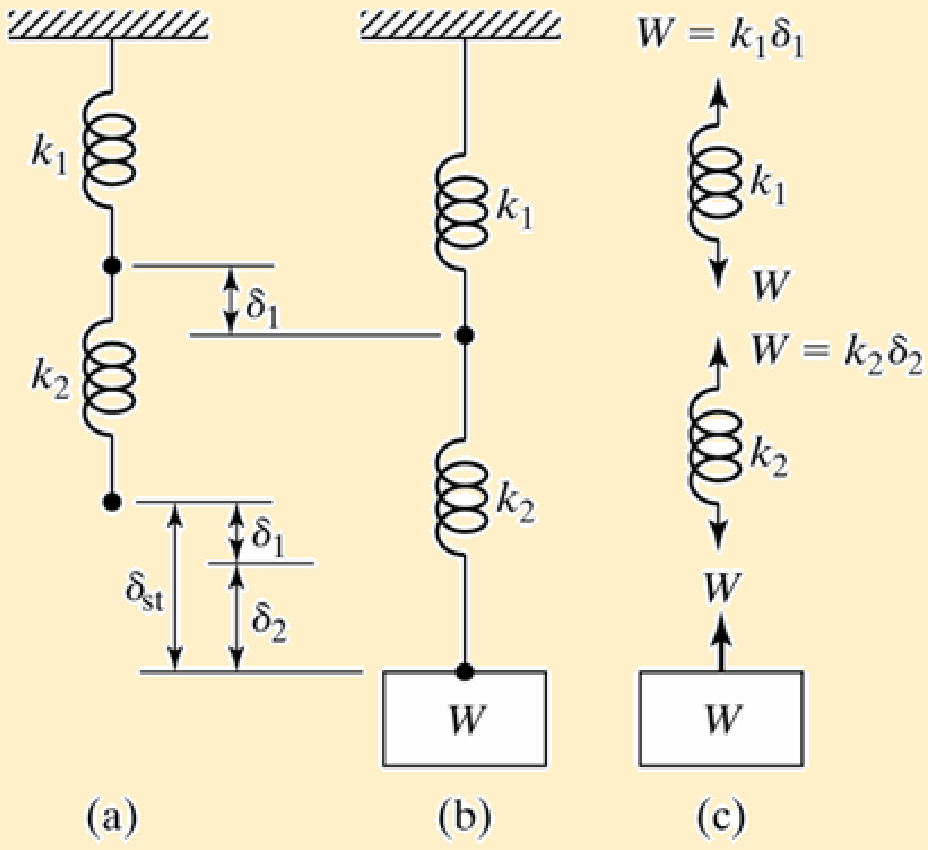


$$k_{eq} = k_1 + k_2$$

Fundamentals concepts

STATICS

Spring special case: series connection

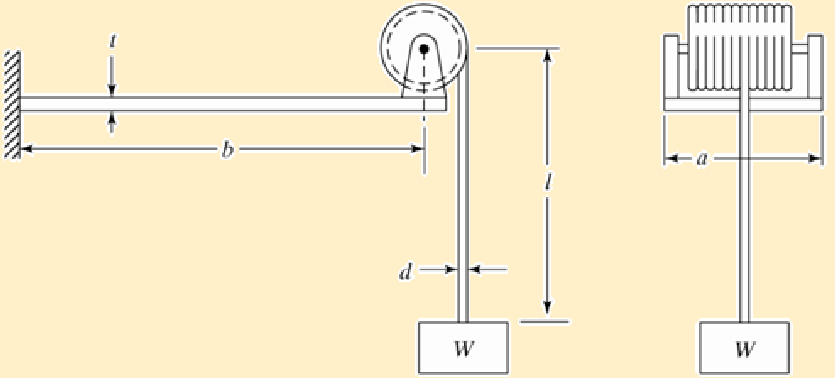


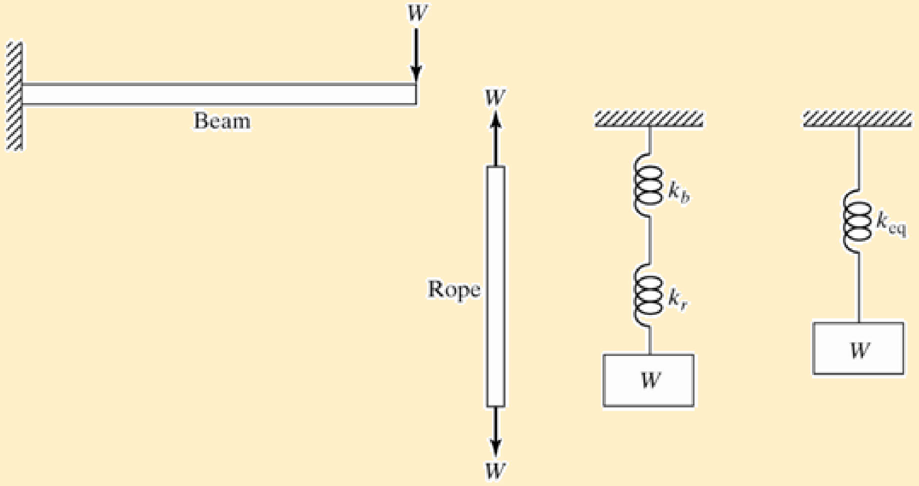
$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Fundamentals concepts

S T A T I C S

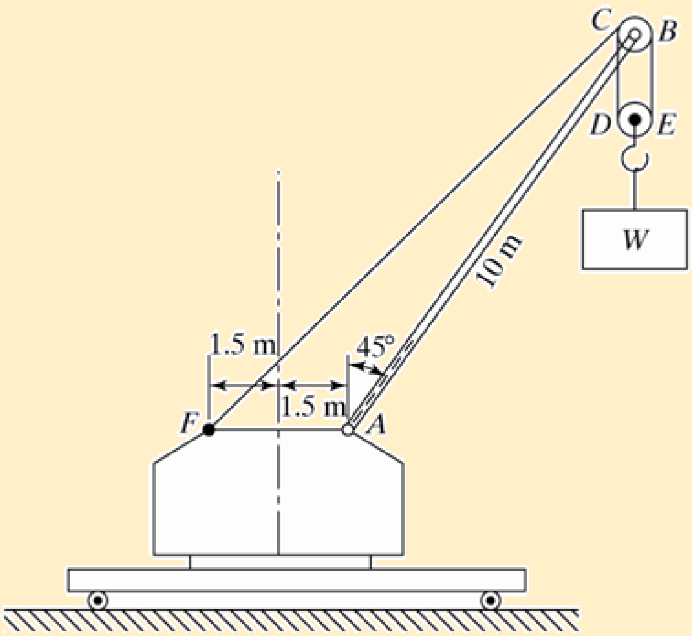
Example

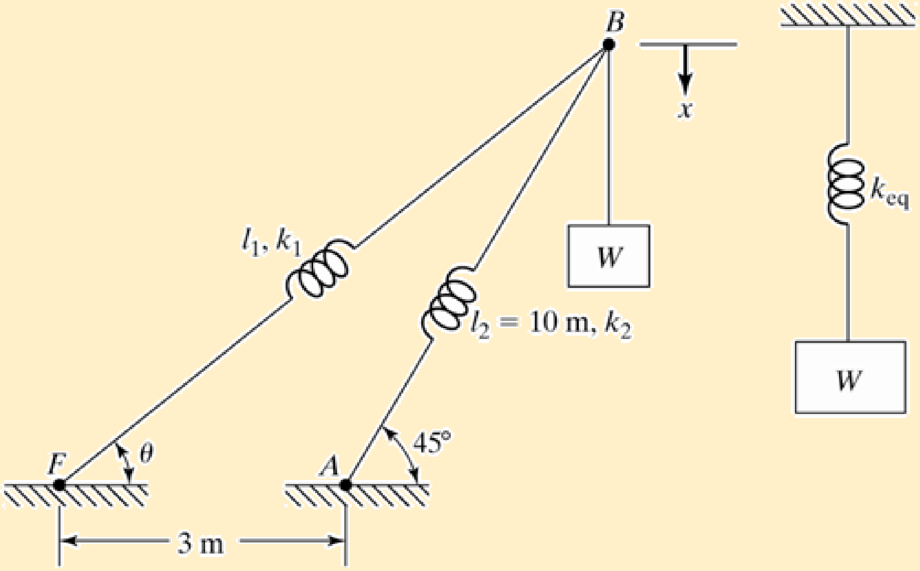




Fundamentals concepts

Example





Fundamentals concepts

S T A T I C S

Vibration system elements : mass or inertia

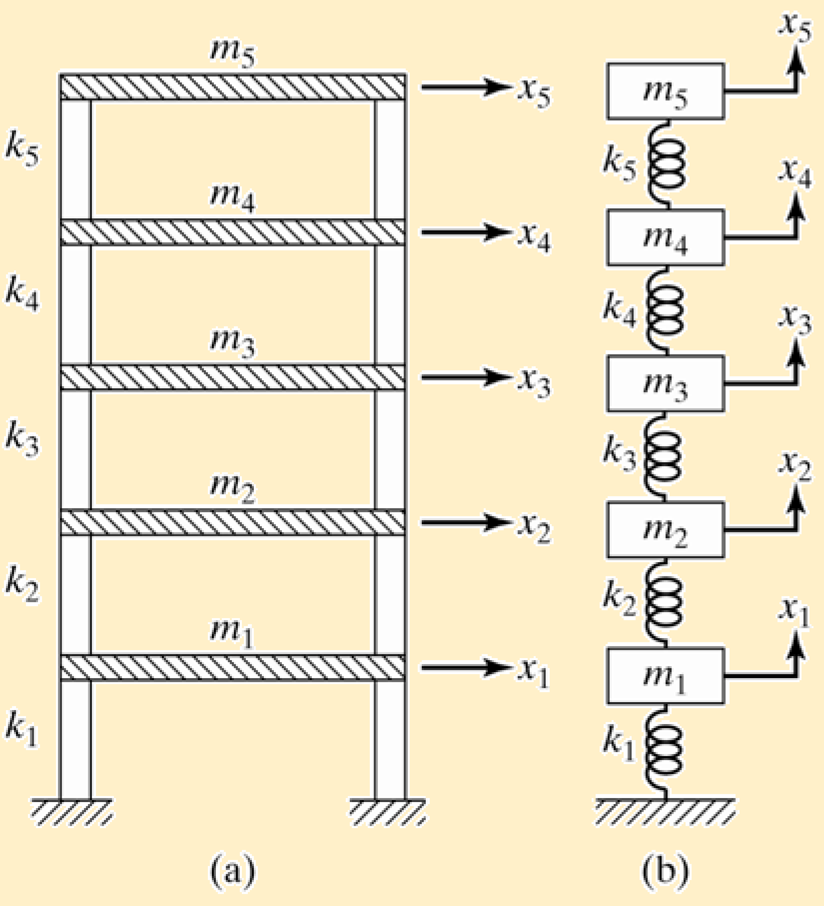
- In translational motion systems, we use the mass, M (kg) .
- In rotational and torsional vibration systems, we use the mass moment of inertia, I ($kg.m^2$).
- Newton's law of motion:

➢ Translational system:

∑forces = Mass * acceleration

➢ Rotational system:

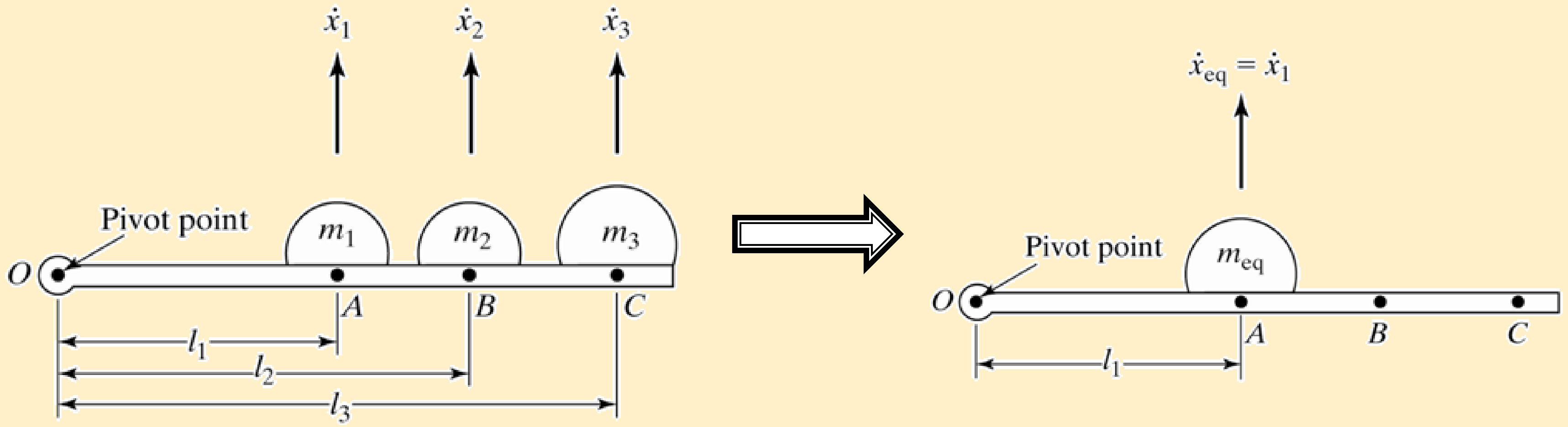
∑Moment = mass moment of inertia *angular acceleration



Fundamentals concepts

S T A T I C S

Case1: translational masses connected to rigid bar

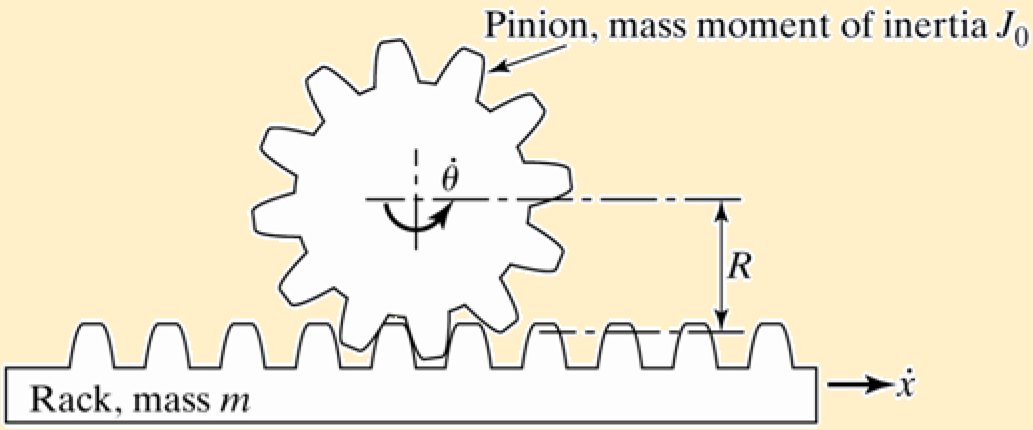


$$m_{eq} = m_1 + \left(\frac{l_2}{l_1}\right)^2 m_2 + \left(\frac{l_3}{l_1}\right)^2 m_3$$

Fundamentals concepts

S T A T I C S

Case2: translational and rotational masses coupled together



Equivalent translational mass

$$m_{eq} = m + \left(\frac{J_o}{R^2}\right)$$

Equivalent rotational mass

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_o \dot{\theta}^2$$

$$J_{eq} = J_o + mR^2$$

Fundamentals concepts

S T A T I C S

Damping effect

- ❖ Viscous damping

❑ Energy dissipation due to motion of mechanical parts in fluids

❑ Amount of dissipated energy depends on:
- ➢ Size and shapes of vibrating bodies
- ➢ Fluid viscosity
- ➢ Vibration frequency
- ➢ Vibrating body velocity

❑ The damping force is proportional to the velocity of the vibrating body

- ❖ Dry friction
- ❖ Material or solid or hysteretic damping

Fundamentals concepts

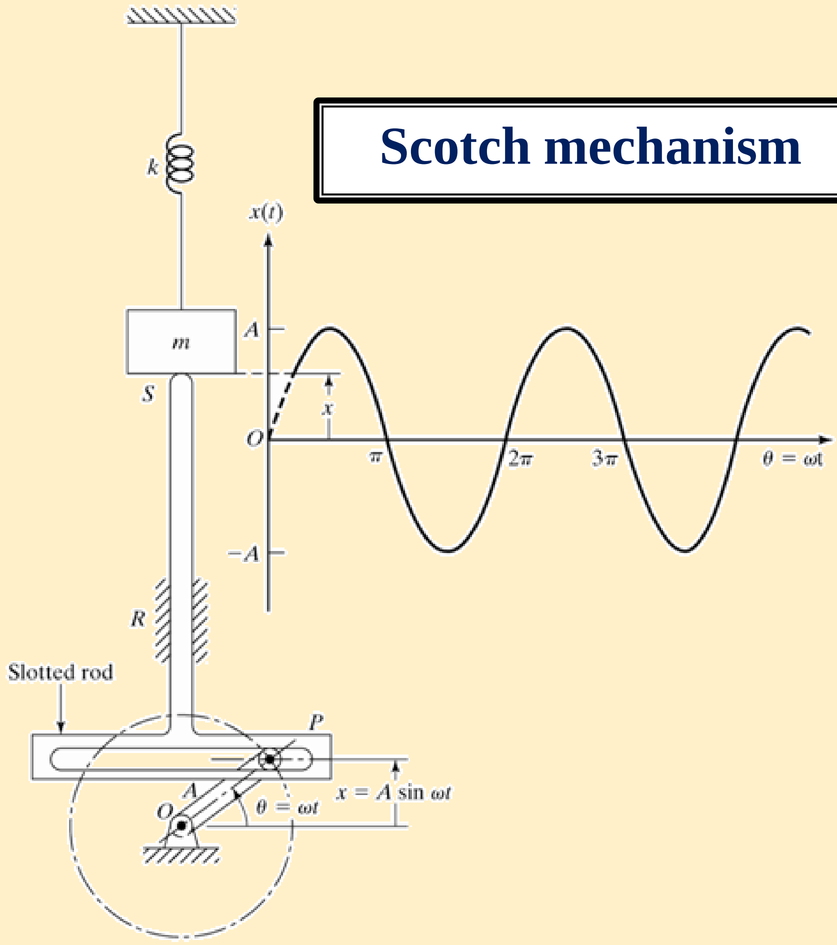
S T A T I C S

Harmonic motion

- Periodic motion is the motion that repeats it self after a period of time
- Harmonic motion is the simplest type of periodic motion
- $x = A\sin(\theta) = A\sin(\omega t)$
- Velocity: $\frac{dx}{dt} = \omega A\cos(\omega t)$
- Acceleration:

$$\frac{d^2x}{dt^2} = -\omega^2 A\sin(\omega t) = -\omega^2 x$$

Scotch mechanism



Fundamentals concepts

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T
A
T
I
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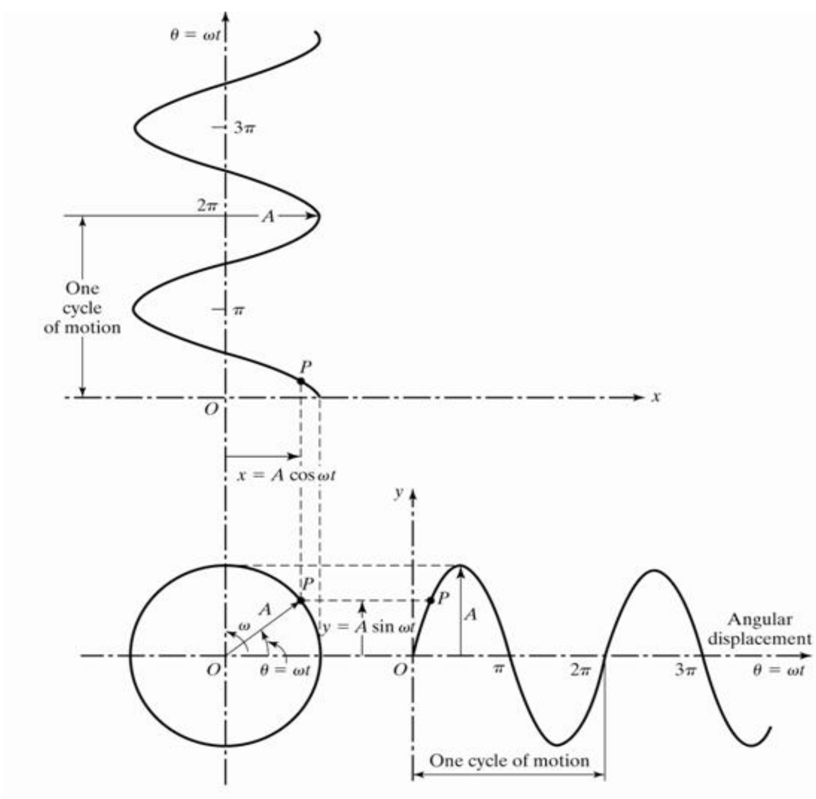
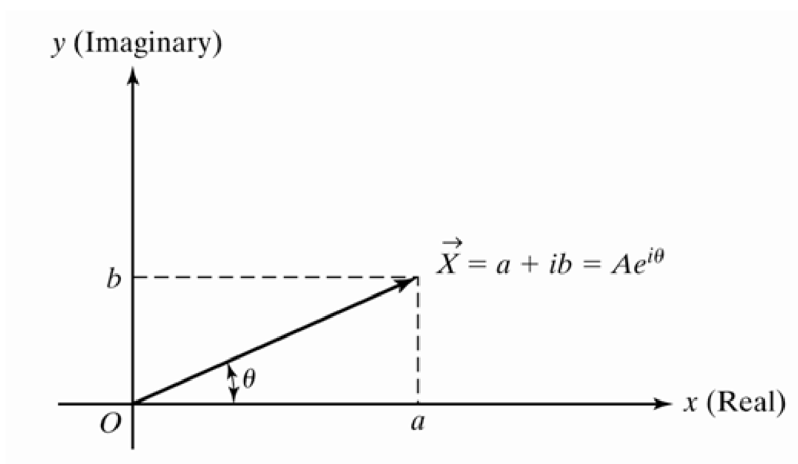
❑Vector notation

y=A sin(ωt)



❑Complex notation

$$\vec{X} = a + bi = A(\cos(\theta) + \sin(\theta)i) = Ae^{i\theta}$$



End of chapter